

Emerging Principles in Fast Trajectory Optimization

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Introduction:

The confluence of major breakthroughs in optimal control theory and computational power has made possible the rapid computation of optimal trajectories. This implies that mission design analysis can be carried out in a quick and efficient manner with the only limitation being the designer's imagination. This course will introduce the student to the major advancements that have taken place over the last decade in both theory and computation that makes fast trajectory optimization possible.

Course Objectives:

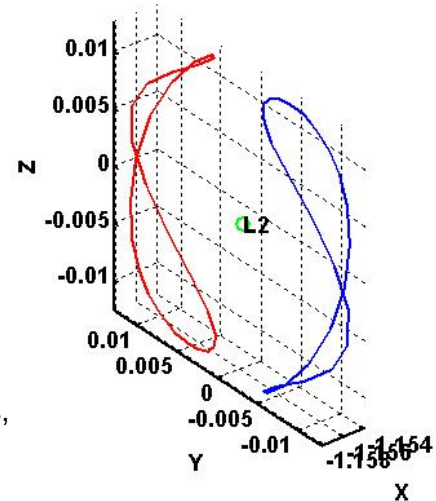
Students will acquire a broad perspective on recent developments in the mathematical foundations of trajectory optimization; "old hats" will also acquire a new perspective to some old ideas. The overall objective of this course is to outline the new foundations related to convergence of solutions that have emerged in recent years and the accompanying breakthroughs in general techniques for problem solving. These techniques are intended to enhance, not replace, special techniques (such as geometric methods) that are in common use. Anyone involved in Astrodynamics research will benefit from this course.

Applications:

Although the course will focus largely on space applications, other applications will also be discussed and elaborated to illustrate the generality of the techniques and ideas. The space application problems that will be illustrated in detail include the following:

1. Finite, continuous and low-thrust trajectory optimization.
2. Mission design for distributed spacecraft systems including formation "flying."
3. Libration point trajectories.
4. Solar sailing and cycler trajectories.
5. Lunar trajectories with applications to the Crew Exploration Vehicle (ORION).
6. Entry, descent and landing trajectory design and guidance.
7. Large-angle spacecraft slew maneuvering.
8. Flexible spacecraft control.

The non-space applications' problem will include such diverse problems as obstacle avoidance of robotic vehicles, unmanned aerial vehicle loitering problems, swing-up of an inverted pendulum and other applications.



Mathematical Background:

Students are expected to have an understanding of basic engineering mathematics at the graduate level. Advanced mathematics concepts required to understand this course, such as the notions of Hilbert and Sobolev spaces, will be introduced and explained with extensive figures and geometric insights. Although students are expected to have a background in elementary optimal control theory (as found in some classic texts like Bryson and Ho) these topics will also be reviewed from a modern perspective.

Course Outline

- I) Overview of recent advances in trajectory optimization
 - 1) A quick look at two tipping points
- II) What kinds of trajectory optimization problems are solvable today and why?
 - 1) A menu of illustrative problems in astrodynamics.
- III) The mathematics of problem formulation
 - 1) Modern terminologies
 - 2) Generic problem formulation
 - 3) Example problems
- IV) Classical optimality conditions
 - 1) Bellman's principle and the curse of dimensionality
 - 2) Pontryagin's principle and the curse of complexity
- V) The Bernoulli-Euler method
 - 1) Euler's original proof of the Euler-Lagrange equations
 - 2) A quick review of Euler's ideas using modern concepts
 - a) Sequence space, uniform convergence and a.e. convergence
 - b) Stone-Weierstrass Theorem
 - c) Passage through the limit – commuting limiting operations
- VI) Karush's extension of the Bernoulli-Euler method
 - 1) Pontryagin's principle via the Karush extension
 - 2) Illustrating hidden convexity
 - 3) The Covector Mapping Principle
 - 4) New first principles in trajectory optimization: A short introduction to Sobolev spaces
- VII) Runge-Kutta (RK) and Pseudospectral (PS) methods for trajectory optimization
 - 1) When and why RK methods fail
 - 2) Introduction to PS techniques: differentiation and integration
 - 3) Weierstrass' theorem, best polynomial and best interpolating polynomial
 - 4) The Bellman PS method
- VIII) Three new modern tests for optimality
 - 1) Quantifying suboptimality
 - 2) Anti-aliasing in optimal control
- IX) Real-time trajectory optimization
 - 1) Lipschitz constant and a mathematical definition of real time
 - 2) Illustrating real-time trajectory optimization
- X) Conclusions and outlook
 - 1) How the new principles avoid dimensionality and complexity curses
 - 2) Current trends in trajectory optimization

Course Instructors

I. Michael Ross

I. Michael Ross is a Professor of Mechanical and Astronautical Engineering, and Program Director of Control and Optimization at the Naval Postgraduate School in Monterey, California. As Program Director, he leads a small group of faculty, research associates, postdoctoral fellows and graduate students towards advancing and applying efficient methods for trajectory optimization. Currently, this program exceeds \$1M per year in research expenditures with generous funding provided by AFOSR, AFRL, NASA, NRO, SSFA and others. He is the principal investigator on several ground and flight experiments involving the design and testing of real-time trajectory optimization methods. He was the Project Lead on PANSAT, a small communications satellite currently in low-Earth-orbit. From 1999 to 2001, he was a Visiting Associate Professor at Draper Labs during which time he introduced pseudospectral methods for solving nonlinear trajectory design and control problems in the areas of launch and entry guidance, attitude control and inertial navigation. An Associate Fellow of the AIAA and the Founding Book Review Editor for the *Journal of Guidance, Control and Dynamics*, his research and teaching interests are in optimal control theory with applications to nonlinear feedback control problems arising in spacecraft formation design and reconfiguration control, low-thrust orbital maneuvers, spacecraft singularity-free control, optimal vibration control, missile guidance and space maneuvers for future systems. Over the last few years, he has taught short courses at Draper Laboratory, NASA and various DoD organizations.

Qi Gong

Dr. Qi Gong received his Ph.D. degree in Electrical Engineering and Computer Science from Case Western Reserve University in 2004. Currently he is a research associate in the Department of Electrical and Computer Engineering at the University of Texas at San Antonio. From March 2004 to March 2005, he was a National Research Council (NRC) postdoctoral researcher at U.S. Naval Postgraduate School (NPS). From March 2005 to May 2006, he was with the Department of Mechanical and Astronautical Engineering at NPS as a research associate. During this time he co-developed theorems on the convergence of pseudospectral (PS) methods for optimal control that included an extension of PS optimal control to non-polynomial basis functions. Dr. Gong's research interests include computational optimal control, trajectory optimization and motion planning, robust and adaptive control of nonlinear systems, real-time optimal control and their industry applications. He has more than thirty publications in these research areas, and has been a frequent invited speaker at several universities and companies. Dr. Gong received the Research Associateship Award from the National Research Council in 2004.